(PMAT 301)

SRR & CVR Government Degree College (Autonomous): Vijayawada

February, 2022-BATCH 2020-21

Department of Mathematics

TOPOLOGY

Semester -III: End Examination

Time: 3 Hours Max. Marks: 60

SECTION -A

Answer any 5 Questions out of 10 Short Answer Questions **5X4=20M**

1. Define Metric space.
2. Let T_1 and T_2 be two topologies on a non-empty set X. and show that $T_1 \cap T_2$ is also a topology.
3. Define Topological space.
4. State Heine –Borel theorem.
5. Define compact space
6. Define Sequentially compact and Totally bounded.
7. Define Hausdorff's space.
8. Define connected space.]
9. State Tietze- Extension theorem.
10. Write about Normal space and Regular space.

SECTION -B

Answer ALL Questions 5X8=40M

11. a) State and prove Baire's Cateogry theorem.

(OR)

b) Let X be a metric space. Then prove that

(i) A subset F of X is closed its complement F' is open.

(ii) Each Open sphere is a Open set.

(PTO)

12. a) Let X be a topological space and A subset of X, then

i) $\bar{A} = A \cup D(A)$ and

ii) A is closed $\Leftrightarrow$ A $\supseteq$ D (A)

(OR)

b). State and Prove Lindelof's theorem

13. a). State and Prove Tychonoff's Theorem.

(OR)

b). Any continuous image of a compact space is compact.

14. a). State and Prove Urysohn's lemma.

(OR)

b) The product of any non-empty class of Hausdorff's spaces is a. Hausdorff's space.

15. a) Any continuous image of connected space is connected.

(OR)

b). Let X be a Hausdorff's space. If X has an open base whose sets are also closed, then show that X is totally disconnected

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(PMAT 302)

SRR & CVR Government Degree College (Autonomous): Vijayawada

February, 2022-BATCH 2020-21

Department of Mathematics

PROBABILITY & STATISTICS

Semester -III: End Examination

Time: 3 Hours

Max. Marks: 60

Answer any 5 Questions out of 10 Short Answer Questions. **5x4=20M**

1. Show that $P(E^c) = 1-P(E)$.
2. In a single throw with two dice find the probability of throwing a sum 10.
3. Two coins are tossed simultaneously. Let X denote the number of heads. Find mean and variance.
4. Define Types of random variables.
5. Derive the Mean of the Poisson distribution.
6. Define Binomial distribution.
7. Define Correlation.
8. Write formula for Rank correlation co-efficient.
9. Define Chi-square distribution of goodness of fit.
10. Write formula T-test for single mean.

Answer ALL Questions **5x8=40 Marks**

11. (a). A bag contains 4green, 6black and 7white balls. A ball is drawn at random. What is the probability that is either a green or a black ball?.

(or)

(b). State and prove Baye's theorem.

12. (a). Write the properties of Characteristic function.

(or)

(b). A random variable 'X' has the following probability function

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine (i) k (ii) Evaluate $P(X < 6), P(X \geq 6), P(0 < X < 5)$.

(PTO)

13. (a). Fit a Binomial distribution of the following data

x	0	1	2	3	4	5
f(x)	2	14	20	34	22	8

(or)

(b). For the continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} cx(2-x), if\ 0 \le x \le 2 \\ 0\ otherwise \end{cases}$$ Where c is a constant. Find 'c', Mean and Variance of X.

14. (a). A random sample of 5 college students is selected and their grades in Mathematics and Statistics are found to be

	1	2	3	4	5
mathematics	85	60	73	40	90
statistics	93	75	65	50	80

Calculate Pearman's rank correlation co-efficient.

(or)

(b). Calculate the regression equations of Y on X from the data given below, taking deviations from actual means of X and Y.

Price(Rs)	10	12	13	12	16	15
Amount Demanded	40	38	43	45	37	43

Estimate the likely demand when price is Rs 20

15. (a). A die is thrown 264 times with the following results. Show that the die is baised

(Given $\chi^2_{0.05}$ = **11.07 for 5 d.f)**

No.appeared on the die	1	2	3	4	5	6
frequency	40	32	28	58	54	52

(or)

(b). The nicotine contents in milligrams in two samples of tobacco over found to be as follows by using F-test.

Sample A	24	27	26	21	25	---
Sample B	27	30	28	31	22	36

(PMAT 303)

SRR & CVR Government Degree College (Autonomous): Vijayawada

February, 2022-BATCH 2020-21

Department of Mathematics

GALOIS THEORY

Semester -III: End Examination

Time: 3 Hours

Max. Marks: 60

Section-A

Answer any 5 questions out of the 10 short answer questions **5x4=20M**

1. If $\{N_i\}_{i\in\Delta}$ is a family of R-submodules of an R-module M, then $\bigcap_{i\in\Delta} N_i$ is also an R-submodule of M.
2. For any module M over a ring R, prove that $\{a \in R / aM = (0)\}$ is an ideal of R.
3. Let $f(x) \in F[x]$ be a polynomial of degree >1. If $f(\alpha) = 0$ for some $\alpha \in F$, then f(x) is reducible over F.
4. Show that $x^2 - 2$ is irreducible over Z.
5. Let $f(x) \in F[x]$ be a polynomial of degree ≥ 1 with α as a root. Then α is a multiple root $\Leftrightarrow f'(\alpha) = 0$.
6. Let E be a finite extension of a finite field F. Then $E = F(\alpha)$ for some $\alpha \in E$.
7. If E is a finite separable extension of a field F, then $|G(E/F)| \leq [E:F]$.
8. State the Dedikind Lemma.
9. What is Cyclotonic polynomial. Explain with an example.
10. If a and b are constructible numbers, then $a \pm b$ are also constructible.

Section-B

Answer ALL questions **5x8=40 Marks**

11. a) State and prove fundamental theorem of R-homomorphisms.

(OR)

(PTO)

b) Prove that the sub modules of the quotient module $\frac{M}{N}$ are of the form $\frac{U}{N}$, where U is a submodule of M containing N.

12. a) State and prove Einsteins Criterian for the irreducibility of polynomials.

(OR)

b) Define an irreducible polynomial p(x) over field F. Show that $F(x)/\langle P(x)\rangle$ is a field.

13. a) If E is a finite separable extension of a field F, then E is a simple extension of F.

(OR)

b) State and prove Uniqueness of splitting field.

14. a) State and prove fundamental theorem of Galois theory.

(OR)

b) Let H be a finite subgroup of the group of automorphisms of a field E. Then $[E:E_H]=|H|$

15. a) Prove that a polynomial f(x) over a field F is solvable by radicals over F if and only if splitting field E over F has solvable galois group $G(E/F)$.

(OR)

b) Let K be the set of all real numbers constructible from the set Q of rational numbers, Prove that K is a subfields of the field of real numbers and that K contains Square roots of all non negative numbers in K.

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(PMAT 304)

SRR & CVR Government Degree College (Autonomous): Vijayawada

February, 2022-BATCH 2020-21

Department of Mathematics

MATHEMATICAL METHODS

Semester -III: End Examination

Time: 3 Hours

Max. Marks: 60

Section -A

Answer any 5 questions out of the 10 short answer questions 5x4=20

1. Calculate the fourier series of the function $f(x)=\begin{cases}0 \text{ if } -\pi \le x \le 0\\ \pi \text{ if } 0 \le x \le \pi\end{cases}$
2. Explain periodic function with examples.
3. Write the given difference equation in subscript notation $\Delta^3 y_x + \Delta^2 y_x + \Delta y_x + y_x = 0$
4. Solve the difference equation $y_{n+2} - 5y_{n+1} - 6y_n = 0$
5. Find the extremal of the functional $y(x) = \int (y'^2 + 2y)dx.$
6. State and prove first shifting theorem of Laplace transforms.
7. Find $L\{e^{2t} - \sin 4t - t^5\}$.
8. Find $L\left\{\frac{p}{p^2 - a^2}\right\}$.
9. Find the value of $L^{-1}\left\{\frac{1}{(p+1)(p-2)}\right\}$.
10. Find $L^{-1}\left\{\frac{1}{p^2+2} + \frac{p}{p^2-4}\right\}$.

Section - B

Answer All questions **5x8=40 Marks**

(PTO)

11. a) Find the fourier series of the function $f(x) = x \sin x; -\pi \le x \le \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots = \frac{\pi - 2}{4}$.

(OR)

b) Find the fourier sine series of the function $f(x) = \cos x$ on the interval $[0, \pi]$.

12. a) Determine a curve joining the origin with the point A(1,1) whose rotation about the axis of abscissa generates a surface of minimum area.

(OR)

b) Find the extremal of the functional $v(y(x)) = \int_{x_0}^{x_1} \left[x^2 \left(y'^2 \right) + 2y^2 + 2xy \right] dx$.

13. a) Solve the difference equation $y_{n+2} - 5y_{n+1} - 6y_n = 4^n, y_0 = 0, y_1 = 1$.

(OR)

b) (i) Solve $y_{n+2} - 4y_{n+1} + 3y_n = 5^n$.

(ii) Solve $y_{n+2} - 2y_{n+1} + y_n = n^2 2^n$.

14. a) If F(t) is a function of class A and if $L\{F(t)\} = f(p)$ then $L\{t^n . F(t)\} = (-1)^n \frac{d^n}{dp^n} f(p)$

(OR)

b) State and prove Initial - value theorem.

15. a) State and prove Convolution Theorem.

(OR)

b) Using Heaviside's expansion formula find $L^{-1}\left\{ \frac{2p^2 + 5p - 4}{p^3 + p^2 - 2p} \right\}$

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SRR & CVR Government Degree College (Autonomous): Vijayawada

M.Sc. DEGREE EXAMINATION, FEBRUARY, 2022

(Examination at the end of III Semester)

MATHEMATICS

ANALYTICAL NUMBER THEORY

(Effect from the admitted batch 2020 - 2021)

Time: 3 Hours Max. Marks: 60

Answer any 5 Questions out of 10 Short Answer Questions **5X4=20M**

1. If $n \geq 1$ then prove that $\sum \mu(d) \left[\frac{1}{n}\right] = \begin{cases} 1 \text{ if } n = 1, \\ 0 \text{ if } n > 1. \end{cases}$
2. Define the mangoldt function $\Lambda(n)$.
3. Prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + O\left(\frac{1}{x}\right)$, where $x \geq 1$.
4. State and prove Legendre's identity.
5. Define Chebyshev's functions.
6. State Abel's identity.
7. Prove congruence is an equivalence relation.
8. State Little Fermat's Theorem.
9. For any prime p, prove that $(p-1)! \equiv -1 \pmod{p}$.
10. Solve the congruence $25x \equiv 15 \pmod{120}$.

Answer Five Questions Choosing One question from each unit.

All questions carry equal marks **5X8=40M**

11. a) Define Euler totient function $\emptyset(n)$. Show that if $n \geq 1$ then we have $\emptyset(n) = \sum_{d|n} \mu(d) \frac{n}{d}$.

(OR)

b) Let f be multiplicative. Then f is completely multiplicative if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

(PTO)

12. a) State and prove Euler's summation formula.

(OR)

b) For $x \geq 1$, prove that $\sum_{n \leq x} \mu(n) \left\lfloor \frac{x}{n} \right\rfloor = 1$ and $\sum_{n \leq x} \Lambda(n) \left\lfloor \frac{x}{n} \right\rfloor = \log \lfloor x \rfloor !$.

13. a) Show that for every integer $n \geq 2$ we have $\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$.

(OR)

b) Show that there is a constant A such that $\sum_{p \leq x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right)$ for all $x \geq 2$.

14. a) State and prove Euler- Fermat theorem.

(OR)

b) State and prove that Lagrange's Theorem.

15. a) State and prove Chinese Remainder Theorem.

(OR)

b) The set of lattice points in the plane visible from the origin contains arbitrarily large gaps. That is, given any integer k > 0 there exists a lattice point (a, b) such that none of the lattice points $(a + r, b + s), 0 < r \leq k, 0 < s \leq k$, is visible from the origin.

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